1. Computational problems:

1. Gaussian elimination: Determine the values of b and c for which the following system will have i. no solution, ii. infinitely many solutions and iii. an unique solution.
2. Determine if a given subset S of a vectorspace V form a subspace of V. If so, find a basis for S. If not, find the subspace spanned by S.
3. Find the matrix of a given linear transformation with respect to your chosen bases.
4. Find the matrix of a given linear transformation $L : V \to W$ with respect to a given basis B of V and C of W.
5. Given a matrix A, compute the rank of A, a basis for the row space of A, and a basis for the column space of A.
6. Find the transition matrix (change of basis matrix) from a one basis for V to another.
7. Determine if a given set of vectors is linearly independent.
8. Determine if a given set is an orthonormal basis.
9. Find the characteristic polynomial and eigen values of a matrix.
10. Determine if a given λ is an eigen value of a given matrix.
11. Find one eigen vector and a basis for the eigen space of a matrix A corresponding to a given eigen value.
12. Given the eigen values or the characteristic polynomial of A, write all possiblities for the Jordan normal form of A.
13. Compute a basis for the kernel and the image of a linear transformation.
14. Determine if a given square matrix is diagonalizable over R = real numbers
15. Compute an othonormal basis that diagonalizes a given linear transformation.
16. Use Gram-Schmidt process to convert a given basis into an orthonormal basis.
17. Cramer's Rule: Use Cramer's rule to solve the given system of linear equations.
18. Find a basis for the orthogonal complement of a given subspace in R^n.
19. Find all possible $n \times n$ matrices that commute with a given $n \times n$ matrix. Find a basis and the dimension of this subspace of $R^{n\times n}$.
20. Find the least squares solution of the given system.

2. Give Examples of:

1. A basis for P_6
2. A subspace of R^7 of dimension 5.
3. An inconsistant system of linear equations in 4 variables.
5. A linear transformation from R^5 to R^{2x3}.
6. Two matrices with $x^2 + 6x + 5$ as the characteristic polynomial.
7. Two 3 x3 matrices with 5 as the only eigen value.
8. A vector space of dimension 3.
9. A non invertible square matrix.
10. A 4 x 5 matrix whose column space has dimension 3.
11. A subspace of P_3 of dimension 2 and containing the vector $X^2 - 1$.
12. A 3×3 matrix that is not diagonalizable.
13. A 4×4 matrix that is not diagonal but is diagonalizable.
14. A 2×2 matrix that is diagonalizable over the complex numbers but not over the real numbers.
15. Two different inner products for P_4.

3. Some short questions and theorems.

1. Prove that any set of mutually orthogonal non zero vectors is linearly independent.

2. Prove that similar matrices have the same characteristic polynomial. Give examples to show the converse is not true, that is, two matrices with the same characteristic polynomial need not be similar.

3. Prove that a matrix A is invertible if and only if zero is not an eigen value of A.

4. V is a subspace of R^7. W_1 and and W_2 are subspaces of V.

W_1 is orthogonal to W_2. dim $W_1 = 3$ and dim $W_2 = 3$.
What is (are) the possible dimension(s) for V? Why?

5. Let $\mathbf{v_1}, \mathbf{v_2}, \cdots, \mathbf{v_n}$ be linearly independent vectors in a vector space V. Show that $\mathbf{v_2}, \cdots, \mathbf{v_n}$ cannot span V.

6. Suppose that λ is an eigen value of A. Show that λ is also an eigen value of A^t.

7. Show that if V and W are two vector spaces of the same dimension, then they are isomorphic to each other. Give an example of two distinct vector spaces of dimension 4 and exhibit an isomorphism between them.

8. Prove that if A is orthogonally diagonalizable then A must be symmetric.

10. A linear transformation $L : V \to W$ is determined by its values on a basis of V. That is, If $B = \{\mathbf{v_1}, \mathbf{v_2}, \cdots \mathbf{v_n}\}$ is a basis for V, and $\mathbf{w_1}, \mathbf{w_2}, \cdots \mathbf{w_n}$ are n vectors in W, then there exists a unique linear transformation $L : V \to W$ such that $L(\mathbf{v_i}) = \mathbf{w_i}, 1 \le i \le n$.

11. Write down six statements for a matrix A that are equivalent to " A is invertible".

12. A is a 3×3 matrix. It is diagonalizable. It has only one eigen value, namely 2. Can you say what A is? why?

13. Show that any 2×2 skew symmetric matrix is diagonalizable over Complex numbers, but not over real numbers unless it is the zero matrix.

14. Is it true that all inner product spaces over real numbers are isomorphic to R^n with dot product? Why?