Name:

331WS96 Final exam

Show all your work. Maximum possible is 161. There are SIX pages.

$R^{m\times n}$ denotes the vector space of all $m \times n$ matrices over the real numbers R. P_n denotes the vector space of all polynomials over the real numbers of degree **less than** n.

1.(24)a. $L : R^3 \to P_4$ is a linear transformation given by
$L((a_1, a_2, a_3)^t) = a_3x^3 + (a_2 - 3a_1)x^2 + a_3x - 2a_1$
Write the matrix of L with respect to the usual basis of R^3 and your chosen basis for P_4.(YOU MUST SPECIFY YOUR CHOSEN BASIS for P_4)

b.. $A = (a_{ij})$ is a 4 x 4 matrix, where $a_{ij} = i^2 - j^2$. Write down A and compute its determinant.

c. What are the possible ranks of a linear transformation $L : R^{2\times 3} \to P_4$? What are the possible dimensions for the kernel of L? Justify your answers.

2.(24) a. A is a 3 x 3 matrix over the real numbers with eigen values -3, 2 and 5. What can you say about A? Justify your answer. Does it follow that A is diagonalizable? Does it follow that A is symmetric? Does it follow that A is invertible? Is it possible for A to be orthogonal?

b. Prove or disprove: A square matrix A is orthogonal if and only if A^2 is orthogonal.

c. Prove or disprove: If the columns of an $n \times n$ matrix A span R^n, then the rows are linearly independent.

3.(21)a. Define: A linear Transformation:

b. Determine if the following function $L : P_4 \to R^3$ is a linear transformation: $L(f(x)) = [f(0), f(-1), 2f(2)]^t$

c. Give an example of a linear transformation from $R^5 \to P_5$ that is not one-one.

4.(15 points) Find all 'a' such that $\{x^2 + 2ax + 2, ax^2 + x + a, 3x - 1\}$ form a basis for the vector space of polynomials of degree less than or equal to 2. Justify your answer.

5.(22) Determine if the following matrices are diagonalizable over (i) the real numbers and (ii)over the complex numbers. Justify your answers.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 4 & -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 & 0 & 1 \\ -1 & 1 & 3 & 2 \\ 0 & 3 & 5 & 1 \\ 1 & 2 & 1 & \pi \end{bmatrix}, C = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 5 \end{bmatrix}$$

6.(31) a. Define: Eigenvalue of a matrix.

b. Prove that λ is an eigen value of A if and only if $A - \lambda I$ is not invertible.

c. Find an orthogonal matrix X such that $X^{-1}AX$ is diagonal:

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

7.(14) Determine all possible matrices A up to similarity if the characteristic polynomial $P_A(x) = (x-2)^2(x-3)^3$

8.(10) STATE **completely** two of your favorite or the most important theorems you have learnt in this class. no credit for stating 2 or 6b from this test.]