

The quartic integrability and long time existence of steep water waves in 2d

Sijue Wu

University of Michigan

Abstract

Abstract: It is known since the work of Dyachenko & Zakharov in 1994 that for the weakly nonlinear 2d infinite depth water waves, there are no 3-wave interactions and all of the 4-wave interaction coefficients vanish on the resonant manifold. In this talk I will present a recent result that proves this partial integrability from a different angle. We construct a sequence of energy functionals $\mathfrak{E}_j(t)$, directly in the physical space, that involves material derivatives of order j of the solutions for the 2d water wave equation, so that $\frac{d}{dt}\mathfrak{E}_j(t)$ is quintic or higher order. We show that if some scaling invariant norm, and a norm involving one spacial derivative above the scaling of the initial data are of size no more than ε , then the lifespan of the solution for the 2d water wave equation is at least of order $O(\varepsilon^{-3})$, and the solution remains as regular as the initial data during this time. If only the scaling invariant norm of the data is of size ε , then the lifespan of the solution is at least of order $O(\varepsilon^{-5/2})$. Our long time existence results do not impose size restrictions on the slope of the initial interface and the magnitude of the initial velocity, they allow the interface to have arbitrary large steepnesses and initial velocities to have arbitrary large magnitudes.