

**ERRATA OF “LOCAL MONOMIALIZATION AND
FACTORIZATION OF MORPHISMS”**

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We list here errata from our paper “Local monomialization and factorization of morphisms” [5].

page 39, line 28: “ $f_{ij} \geq 0$ for all i, j ” should be “ $f_{ij} \geq 0$ for $0 \leq i, j \leq s$ ”.

page 45, line 15: “ $\overline{T}'(1)$ has regular parameters” should be “ $\overline{T}(1)$ has regular parameters”.

page 49, line 26: “ $s \leq m \leq l$ ” should be “ $s \leq m \leq n$ ”.

page 58, lines 19-22: Replace “Set $\sigma(i)$ to be the largest possible $\sigma(i) \geq s$ ” with “Set

$$\sigma(i) = \dim k(\overline{T}(i))[[\bar{z}_1(i), \dots, \bar{z}_m(i)]]/p_m(i)$$

where $p_m(i)$ is defined in (53). We have $\sigma(i+1) \leq \sigma(i)$ for $0 \leq i \leq t-1$ in (45)” The inequality $\sigma(i+1) \leq \sigma(i)$ is proven in Lemma 6.3 of [6].

page 62, line 8: “By (42)” should be “By (43) and (43)”.

page 67, line 2: “ $u(\tilde{y}_1(t), \dots, \tilde{y}_l(t))$ ” should be “ $\Sigma(\tilde{y}_1(t), \dots, \tilde{y}_l(t))$ ”.

page 73, line 1: “ $g_{ij} \geq 0$ for all i, j ” should be “ $g_{ij} \geq 0$ for $0 \leq i, j \leq s$ ”.

page 73, line 2: “ $U''(\alpha)[N_1, \dots, N_s, N_r]$ ” should be “ $U''(\alpha)[N_1, \dots, N_s, N_r, \frac{1}{N_r}]$ ”.

page 73, line 6: After “in an algebraic closure of $k(U(\alpha+1))$ ” insert “if $g_{s+1, s+1} > 0$.”

$$M_r - c_{\alpha+1} = \prod_{i=1}^{-g_{s+1, s+1}} (N_r^{-1} - \omega^i d_{\alpha+1}^{-1}) \beta_r$$

where ω is a primitive $-g_{s+1, s+1}$ -th root of unity if $g_{s+1, s+1} < 0$.”

page 73, line 7: “ $U''(\alpha)[N_1, \dots, N_s, N_r]$ ” should be “ $U''(\alpha)[N_1, \dots, N_s, N_r, \frac{1}{N_r}]$ ”.

page 93, line 3: “ $\Omega \in m(U''(0))^N$ ” should be “ $\Omega \in m(U(0))^N$ ”.

page 93, after line 7, insert:

“(5): Suppose that $g = \bar{y}_1^{d_1} \cdots \bar{y}_s^{d_s} \Sigma(\bar{y}_1, \dots, \bar{y}_l) + \Omega$ where $\nu(\bar{y}_1^{d_1} \cdots \bar{y}_s^{d_s}) > A$ and $\Omega \in m(U(0))^N$ with $N\nu(m(U(0))) > \nu(\bar{y}_1^{d_1} \cdots \bar{y}_s^{d_s})$. Then there exists a

CRUTS along ν as in the conclusions of Theorem 4.9 such that

$$g = \bar{y}_1(t')^{d_1(t')} \cdots \bar{y}_s(t')^{d_s(t')} \bar{\Sigma}(\bar{y}_1(t'), \dots, \bar{y}_l(t'))$$

where $\nu(\bar{y}_1(t')^{d_1(t')} \cdots \bar{y}_s(t')^{d_s(t')}) > A$

page 95, lines 26-31: “Let G be $g \in k(c_0)[[\bar{x}_1(1), \dots, \bar{x}_l(1)]][x_{l+1}]$ ” should be “Since

$$\bar{y}_i^d = \bar{x}_1^{f_{i1}} \cdots \bar{x}_s^{f_{is}} \phi_1^{-f_{i1}} \cdots \phi_s^{-f_{is}}$$

for $1 \leq i \leq s$, by Lemma 4.2, we can perform a MTS of type (M1) to get $g' \in k(U''(0)[[\bar{x}_1(1), \dots, \bar{x}_l(1)]][x_{l+1}]$.

Let G be the Galois group of a Galois closure of $k(U''(0))$ over $k(c_0)$. Since x_{l+1} is analytically independent of $\bar{y}_1^d, \dots, \bar{y}_s^d, \bar{y}_{s+1}, \dots, \bar{y}_l$ (by Theorem 2.12) we can define

$$g = \prod_{\tau \in G} \tau(g')$$

where G acts on the coefficients of g' . We have $g \in k(c_0)[[\bar{x}_1(1), \dots, \bar{x}_l(1)]][x_{l+1}]$ ”

page 115, line 5 of the statement of Theorem 5.3: “if m_V is the maximal ideal of V and $p^* = m_V \cap S$ ” should be “if q is a prime ideal of V and $p^* = q \cap S$ ”.

page 115, line 6 of the statement of Theorem 5.3: “segments” should be “isolated subgroups”.

page 121, line 17: “ $t > \max\{a_{ij}, g_{ij}(1)\}$ ” should be “ $t > \max\{a_{ij}, g_{ij}(r)\}$ ”.

page 122, lines 4-5: “if m_V is the maximal ideal of V and $p^* = m_V \cap S$ ” should be “if q is a prime ideal of V and $p^* = q \cap S$ ”.

page 122, lines 19-20: “if m_V is the maximal ideal of V and $p^* = m_V \cap S$ ” should be “if q is a prime ideal of V and $p^* = q \cap S$ ”.

page 122, line 6 of the statement of Theorem 5.5: “segments” should be “isolated subgroups”.

page 6, line 31: After “Theorem 1.6” insert:

“There is however uncertainty about the result of [9] (c.f. [1], [2]). There is a local version of this result for morphisms of toric varieties which has been proven. We may use this result instead of [9] to obtain proofs of Theorems 1.9 and 1.10 below. Christensen [3] has proven local strong factorization of morphisms of toric 3-folds along a toric valuation. Using the language of toric geometry, Karu [8] has extended Christiansen’s result to prove this local result in all dimensions. A proof in the spirit of Christensen’s original proof, using only elementary properties of determinants, is given in [7].”

page 136, line 10: After “Theorem 7.1”, insert: “A proof of Theorem 1.9 using Theorem 1.1 (Local Monomialization) and Local Strong Factorization of morphisms of toric varieties along a toric valuation ([3], [8], [7]) which is thus independent of [9] is given in Theorem 3.3 [7]. A proof of Theorem 7.1 which is independent of [9] is now immediate from this proof of Theorem 1.9”

Errata of "Local monomialization of transcendental extensions"

page 1525, line 9 " $\nu(f_{I_0}) > \nu(f_I)$ " should be " $\nu(f_I) > \nu(f_{I_0})$ ".

page 1525, line 10 " $\nu(g_{J_0}) > \nu(g_J)$ " should be " $\nu(g_J) > \nu(g_{J_0})$ ".

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